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Oligopolistic Non-Linear Pricing and Size Economies

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Abstract

The effects of non-linear pricing are determined by the relationship between the demand and the technological structure of the market.

This paper focuses on a model in which firms supply a homogeneous product in two different sizes. Information about consumers' reservation prices is incomplete and the production technology is characterized by size economies.

Four equilibrium regions are identified depending on the relative intensity of size economies with respect to consumers' evaluation of a second unit of the good. The desirability of non-linear pricing varies across different equilibrium regions.

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1 Introduction

Production technology plays an important role in determining the effects of non-linear pricing in imperfectly competitive settings. It is very well known that the possibility of practicing non-linear pricing and its implementation by a monopolist firm depend crucially on the characteristics of the market demand, on the information available, and on the instruments the firm can use to implement a discriminatory price strategy. As shown by Stole[16], similar observations apply when the market structure is competitive although additional strategic effects have to be taken into account. The demand side, however, is not the only factor determining the feasibility and the effects of non-linear pricing. This aspect is underlined by Ekelund[9]: he observes that Jules Dupuit was already aware in 1849 of the strong impact that both the preference structure and the technology available for production play in influencing the possibilities of firms to price discriminate. Despite this, most of the related theoretical literature does not emphasize the interrelationships between the demand and the cost structure in determining firms' pricing strategies¹.

The main contribution of this paper is to explicitly take into account the interaction between the demand structure and the production technology when firms are price discriminating via non-linear pricing.

Technological factors play an important role in regulated industries: natural monopolies and network utilities are typical examples. Another characteristic of these industries is the asymmetric information often faced by the regulatory agency. Non-linear pricing is a more efficient device to regulate an industry with those characteristics and the technological structure influences the shape of the price schedule. This paper, however, focuses on purely profit maximizing firms in a *unregulated* sector. The relation between the effects of

¹The influential survey by Varian[19] largely assumes that firms produce at a constant marginal cost, often normalized to zero. More recent literature reviews by Armstrong[1] and Stole[17] share the same feature.

quantity discounts and the technological structure of the market constitutes an old topic in microeconomic theory. Quantity discounts have a positive effect on the profits of a monopolist and hurt consumers; the effects of this practice are not as clear when firms are competing in an oligopolistic market. Ireland[11]’s results suggest that non-linear pricing always hurts consumers and welfare, unless size or scale economies are sensibly important. Using the same model, however, Cheung-Wang[4] show that Ireland’s conclusion may be too strong and there are cases in which firms practicing linear pricing may restrict their output at consumers’ expenses. On the other hand, De Meza[7] proves that in a differentiated Bertrand duopoly model non-linear pricing may, instead, enhance consumers surplus. The debate on competition, technological structure and the effects of non-linear pricing is then far from settled.

The objective of this paper is tackle the above issues focusing on firms competing ‘à la Cournot’. In the model, firms do not possess information on consumers’ preferences but only on their distribution. The good is supplied in packets of two different sizes and firms’ enjoy size economies when producing and distributing the two-unit packets². Under specific conditions on the distribution function, it is possible to get a closed form solution and so analyze directly the results in terms of consumers served, output supplied, consumers surplus, profits and welfare. A further advantage of the model presented this paper is that it makes the comparison of the properties of equilibria under linear and non-linear pricing particularly easy. This is in sharp contrast with more general models of non-linear pricing in which, if the comparison is possible, requires more elaborate analysis³. Moreover, the analysis provided is directly comparable with the ones of Ireland[11] and Cheung-Wang[4].

²Focusing on packets of two different sizes only does not harm the generality of the findings: the model is easily extended to firms supplying the good in m -sizes packets.

³A recent example of a general model from which is possible to carry out a comparison of non-linear and linear pricing is Armstrong-Vickers[2].

The findings of the paper are not just limited to the case of bundling and size economies. First, a strikingly similar modelling approach has been adopted to address a number of issues in the literature on non-linear pricing: Deneckere-Mc Afee[8], for example, adopt a closely related specification for their analysis of damaged goods. Second, both the approach and the results obtained are strongly connected to the literature on competitive quality supply. De Fraja[6] and more recently a series of paper by Johnson-Myatt ([12]-[13]) analyze Cournot models in which firms are allowed to supply ‘product lines’. A productive parallel can be drawn between their interpretation and the size-related discounts analyzed in this paper: our results depend crucially on size economies while theirs on the analogous concept of returns to quality in supply.

The results obtained can be summarized as follows. Firstly, necessary and sufficient conditions are provided for an equilibrium with non-linear pricing to be well-behaved or to collapse to a non-discriminatory situation in which firms provide the good only in packets of two units. An analogous result is obtained when firms practice linear price schedules. This allows us to partition the parameter space: four types of equilibrium configurations with different properties under linear and non-linear pricing are devised. A particularly interesting result is that both consumers surplus and welfare may be enhanced under non-linear pricing.

The paper is structured as follows. Section 2 describes the model and introduces firms’ problem. Section 3 provides sufficient conditions under which a well-behaved equilibrium under linear and non-linear pricing is obtained. A direct consequence of the latter analysis is to provide a partition of the parameters’ space which allows to identify different types of equilibria: in Section 4 those equilibria are described and analyzed by comparing the results under non-linear and linear pricing. Finally, Section 5 discusses the results, draws their implications and suggests future extensions. All proofs can be found in Appendix A.

2 The Model

The model is based on Ireland[11] and it is modified in order to focus on technological issues.

2.1 Supply

On the supply side there are n firms that produce a homogeneous good; the good can be commercialized in packets of different sizes: either one-unit or two-units. Once defined p_1 the price of the single unit and p_2 the price for the double packet, prices are linear if $p_2 = 2p_1$; otherwise prices are non-linear. Assuming consumers are rational, it is clear that under these assumptions, two-units packets are attractive to consumers' eyes if and only if $p_2 \leq 2p_1$.

Technology is characterized by size economies: it displays unit cost savings in producing the two-unit packet with respect to the single unit packet. This property represents the main difference with the model in Ireland[11] and Cheung-Wang[4] but it is crucial for the results described in the rest of the paper.

2.1.1 The Cost Structure and Size Economies

The cost structure in Ireland's model is fairly simple: it is assumed that the total cost is described by the following relation

$$C(q, x_1, x_2, Q_q) = \begin{cases} c D_1(x_1, x_2, Q_1) & \text{if } q = 1 \\ 2c D_2(x_1, x_2, Q_2) & \text{if } q = 2 \end{cases}$$

in which q represents the size, x_q the marginal consumers, indifferent between buying size q or $q - 1$ packet, D_q the demand faced by the firm for the size q while Q_q the demand for the size q faced by the $n - 1$ rival firms, i.e. the residual demand. The hypothesis is that in producing the two-units packets

the unit cost is constant and double of the unit cost c of producing a single package.

In Ireland's model the marginal and average cost are identical and constant with respect to both size and quantity. It is reasonable, in a number of situations, to assume that average cost is declining in the size of the product sold. An intuitive way of justifying this assumption is in relation to the packaging and selling costs afforded by the firms to take the product to the market. It can be thought then that the total cost is a function of three inputs:

$$TC = wl + rk + P_q$$

in which l represents labor and w is its given price, k is capital whose price is r and the size-dependent cost of packaging devices is denoted by P_q . It is clear that the packaging cost component depends on size q and it is assumed that P_q/q is weakly decreasing in q , i.e. $P_1 \geq P_2/2$. This implies that the marginal and average costs of production are still constant with respect to demanded quantity, but they decline with size q . The previous discussion allows us to state Assumption 1:

Assumption 1 *The production of each firm takes place according to the following cost function:*

$$C(q, x_1, x_2, Q_q) = \begin{cases} c D_1(x_1, x_2, Q_1) & \text{if } q = 1 \\ 2\theta c D_2(x_1, x_2, Q_2) & \text{if } q = 2 \end{cases}$$

in which $\theta \in [1/2, 1]$.

The parameter θ has an intuitive interpretation: it can be thought as a measure of the savings in packaging costs related to size, i.e.:

$$\theta = \frac{P_2/2}{P_1}$$

Two limiting cases are encompassed in this description: if P_2 is exactly the double of P_1 the model is the same one as in Ireland while if the cost P is fixed and does not depend on the size of the packet, i.e. when $P_1 = P_2$, the value of θ is equal to $1/2$.

2.2 Demand

Consumers are characterized by a type parameter x that expresses their willingness to pay for the first unit of good. Crucial for the results of the model is the distribution function of willingness to pay of customers, $f(x)$ with $x \in [0, 1]$. This is assumed to be continuous and twice differentiable in x . The cumulative distribution $F(x)$ expresses the fraction of customers with a willingness to pay equal or lower than x . Each consumer can demand either the one unit packet or the two units packet or nothing.

The utility function is:

$$U(q, x) = \begin{cases} 0 & \text{if } q = 0 \\ x - E(q) & \text{if } q = 1 \\ bx - E(q) & \text{if } q = 2 \end{cases}$$

in which $x \in [0, 1]$, $q = 0, 1, 2$ are the units of product bought, $E(q)$ is the expenditure necessary to buy the desired packet. Marginal utility is decreasing: the second unit provides a lower utility to the consumer. This is captured by the parameter b , about which the following is assumed:

Assumption 2 *The marginal decrease of utility in consuming a second unit of the good is the same across all consumers so that the willingness to pay for the second unit in the package is $(b - 1)x$, $b \in [1, 2]$.*

All consumers aim to maximize their utility, i.e. choose q such that:

$$\max\{0, x - p_1, bx - p_2\}$$

is selected.

The marginal consumers, given a set of prices (p_1, p_2) can be identified to construct the demand schedule. Agents of type x_1 are indifferent between buying nothing or one unit if:

$$x_1 - p_1 = 0 \Leftrightarrow x_1 = p_1$$

This condition identifies x_1 , the consumers' type for which the individual rationality constraint is binding. Customers of type x_2 are indifferent between one unit or two units if:

$$(b - 1)x_2 = p_2 - p_1$$

from which follows:

$$x_2 = \frac{p_2 - p_1}{b - 1}$$

identifying the type for which the upper incentive compatibility constraint is binding.

To summarize consumer's choices, then:

$$q = \begin{cases} 0 & \text{if } 0 < x < x_1 \\ 1 & \text{if } x_1 < x < x_2 \\ 2 & \text{if } x_2 < x < 1 \end{cases}$$

from which the total demand is computed as follows:

$$D_q(x_1, x_2, Q_q) = \begin{cases} 0 & \text{if } 0 < x < x_1 \\ F(x_2) - F(x_1) - Q_1 & \text{if } x_1 < x < x_2 \\ 1 - F(x_2) - Q_2 & \text{if } x_2 < x < 1 \end{cases}$$

2.3 Non-Linear Pricing

It is possible now to state the firms' problem. The assumption of quantity setting oligopoly implies, in this context, that each firm chooses the amount of packets of the two different sizes to be produced. This is equivalent to setting total production, while the market establishes the equilibrium price, as determined by the demand schedule. All of this implies that firm's choice of quantities is equivalent to choosing the marginal consumers x_1 and x_2 , as defined above⁴.

⁴Oren-Smith-Wilson[14] and Wilson[20] show that it is equivalent to assume that firms' choice variable is quantity or the consumer's type.

Suppose further, consistent with the idea of Cournot competition, that firms act as monopolists on their segments of residual demand. Define Q_1 and Q_2 the quantities supplied by the $(n-1)$ rival firms for the one unit and two units package respectively.

The profit function is defined by:

$$\pi_i = [F(x_2) - F(x_1) - Q_1] (p_1 - c) + [1 - F(x_2) - Q_2] (p_2 - 2\theta c)$$

From the constraints derived in Section 2.2 follows that $p_1 = x_1$ and $p_2 = (b-1)x_2 + x_1$, this can be written as:

$$\pi_i = [F(x_2) - F(x_1) - Q_1] (x_1 - c) + [1 - F(x_2) - Q_2] [(b-1)x_2 + x_1 - 2\theta c]$$

The first order conditions for a maximum are obtained imposing:

$$\begin{aligned} \frac{\partial \pi_i}{\partial x_1} &= [F(x_2) - F(x_1) - Q_1] - f(x_1)(x_1 - c) + [1 - F(x_2) - Q_2] = 0 \\ \frac{\partial \pi_i}{\partial x_2} &= f(x_2)(x_1 - c) + (b-1)[1 - F(x_2) - Q_2] - f(x_2)[(b-1)x_2 + x_1 - 2\theta c] = 0 \end{aligned}$$

Considerations on the sufficiency conditions for a maximum in the firms problem and their relation with the distribution function are delayed until Section 3.

Imposing that the equilibrium decisions of firms are symmetric⁵, the first order conditions can be written as:

$$\frac{\partial \pi_i}{\partial x_1} = \frac{1 - F(x_1)}{n} - f(x_1)(x_1 - c) = 0 \quad (1)$$

$$\frac{\partial \pi_i}{\partial x_2} = (b-1)\frac{1 - F(x_2)}{n} - f(x_2)[(b-1)x_2 - (2\theta - 1)c] = 0 \quad (2)$$

The last two equations define implicitly and independently x_1^* and x_2^* , i.e. the firms' optimal choices when price discrimination is allowed.

⁵Further discussion about symmetry is provided in the concluding section.

2.4 Linear Pricing

Suppose that, for any reason, firms are constrained to a linear price schedule the restriction $p_2 = 2p_1$ has to be imposed. This implies:

$$p_2 - p_1 = p_1 \Rightarrow x_2 = p_1/(b-1) \text{ and } x_1 = (b-1)x_2$$

The profit function can be written, then, as:

$$\pi_i = \left\{ F \left[\frac{x_1}{(b-1)} \right] - F(x_1) - Q_1 \right\} (x_1 - c) + 2 \left\{ 1 - F \left[\frac{x_1}{(b-1)} \right] - Q_2 \right\} (x_1 - \theta c)$$

or alternatively, using the constraint stated above, as:

$$\pi_i = [(b-1)x_2 - c][F(x_2) - F((b-1)x_2) - Q_1] + 2[(b-1)x_2 - \theta c][1 - F(x_2) - Q_2]$$

Maximizing with respect to x_2 yields the following first order condition:

$$\begin{aligned} & -(b-1)f((b-1)x_2)[(b-1)x_2 - c] - f(x_2)[(b-1)x_2 - (2\theta - 1)c] + \\ & +(b-1)[2 - F(x_2) - F((b-1)x_2) - Q] = 0 \end{aligned}$$

in which $Q = Q_1 + Q_2$ represents the units supplied by the other $(n-1)$ firms. By imposing symmetry the above equation can be written as:

$$\begin{aligned} & -(b-1)f((b-1)x_2)[(b-1)x_2 - c] - f(x_2)[(b-1)x_2 - (2\theta - 1)c] + \\ & +(b-1)\frac{2 - F(x_2) - F((b-1)x_2)}{n} = 0 \end{aligned} \tag{3}$$

The second order conditions for this problem are discussed in the next section. The solution of this equation along with the condition $x_1 = (b-1)x_2$ are denoted by x'_1 and x'_2 and represent the firms' optimal choices under linear pricing.

Finally it is important to notice the relationship between the first order conditions of the two problems. First, express the first order conditions (1), (2) and (3) as functions of x as follows:

$$\begin{aligned}FOC_{NLP1}(x) &= 0 \\FOC_{NLP2}(x) &= 0 \\FOC_{LP}(x) &= 0\end{aligned}$$

The first order conditions, then, are linked by the following relation:

$$(b-1)FOC_{NLP1}[(b-1)x] + FOC_{NLP2}(x) = FOC_{LP}(x) \quad (4)$$

This result is of crucial importance to prove the main properties of the non-linear and linear pricing equilibrium outcomes.

3 Identification of Equilibria

3.1 A Remark on Sufficient Conditions for Maximum Profits

Sufficient conditions for a maximum in the firms' optimization problem deserve careful discussion since they are strictly related to the properties of the distribution function. The analysis of Cheung-Wang[4] witnesses the importance of this aspect: in the context of the original Ireland model, a deeper analysis of the second order conditions allows one to highlight more general insights on non-linear pricing.

Consider first the non-linear pricing equilibrium. Recall that the first order conditions with respect to x_1 and x_2 are independent, so that the elements of the minor diagonal of the Hessian matrix are zero. The second order sufficient conditions for a maximum of the firms' symmetric profit maximization

problem are:

$$\frac{\partial^2 \pi_i}{\partial x_1^2} = -\frac{n+1}{n}f(x_1) - f'(x_1)(x_1 - c) \leq 0 \quad (5)$$

$$\frac{\partial^2 \pi_i}{\partial x_2^2} = -(b-1)\frac{n+1}{n}f(x_2) - f'(x_2)[(b-1)x_2 - (2\theta-1)c] \leq 0 \quad (6)$$

which must be satisfied at a maximum point (x_1^*, x_2^*) . Define $R^*(x) = f'(x)(x - c)/f(x)$ so that condition (5) can be re-stated as:

$$R^*(x_1) \geq -\left(1 + \frac{1}{n}\right)$$

Analogously for condition (6):

$$R^{**}(x) = \frac{f'(x) \left[x - \frac{2\theta-1}{b-1}c\right]}{f(x)}$$

is defined so that the second order condition can be written as:

$$R^{**}(x_2) \geq -\left(1 + \frac{1}{n}\right)$$

On the other hand, when firms practice linear pricing, the sufficient condition for a maximum is given by:

$$\pi_i''(x_2) = -(b-1)^2\{f((b-1)x_2) + f'((b-1)x_2)[(b-1)x_2 - c]\} - f'(x_2) \quad (7)$$

$$\left\{\left(1 + \frac{1}{n}\right)(b-1) + [(b-1)x_2 - (2\theta-1)c]\right\} - \frac{(b-1)^2}{n}f'((b-1)x_2) \leq 0$$

to be satisfied at x_2' .

As for first order conditions, there is a relationship between second order conditions and it can be expressed as:

$$(b-1)^2 SOC_{NLP1}[(b-1)x] + SOC_{NLP2}(x) = SOC_{LP}(x)$$

having defined the second order conditions (5), (6) and (7) as functions of x .

3.2 Non-Linear Pricing Equilibrium

When conditions (5)-(6) hold, then a non-linear pricing equilibrium exists, it is unique and it is described by (1)-(2). As existence and uniqueness of equilibrium are guaranteed, the focus is now on the economic properties of the equilibrium.

A well-behaved equilibrium in this context should respect the following requirements:

$$c < x_1^* < x_2^* < 1$$

Proposition 1 establishes when this is the case. Before that, however, it is necessary to introduce a further piece of notation: Define the following expression as:

$$\psi(b) = 1 - \frac{2-b}{2c}$$

Moreover, define also:

$$\vartheta(b) = \frac{b}{2} + \frac{1 - F\left(\frac{2\theta-1}{b-1}c\right)}{f\left(\frac{2\theta-1}{b-1}c\right)} \frac{(2-b)(b-1)}{2}$$

The main result of this section can be now stated.

Proposition 1 *Suppose $\theta \geq \vartheta(b)$, then a non-linear pricing equilibrium in which $c < x_1^* < x_2^* < 1$ is the outcome of firms' profit maximization problem. If, instead, $\theta \leq \psi(b)$ then the non-linear pricing equilibrium is characterized by $c < x_1^{**} \equiv x_2^{**} < 1$.*

The proposition provides sufficient conditions on the parameters to register a well-behaved equilibrium. In that case both one and two unit packets are supplied, which is exactly the case when the inequality $\theta \geq \vartheta(b)$ holds. However, the proposition is also providing a sufficient condition under which a corner solution is found: if $\theta \leq \psi(b)$, then all firms will supply only two-unit packets and not the one-unit ones.

The intuition for the result in Proposition 1 is that firms in equilibrium choose to supply both sizes packets (one and two units) when the effect of

size economies (θ) is not too intense relative to the consumers' evaluation of a second unit of product (b). In other words, when the cost savings related to size economies are relatively important, firms find it optimal to supply only two-units packets.

The parameter space ($bO\theta$) results then split into several areas by the relations derived: the properties of the equilibrium are different for different combinations of parameters. It is not possible however to completely identify equilibria. For example, it is *a priori* not possible to predict what characteristics the equilibria will have when the parameters satisfy⁶:

$$\psi(b) < \theta < \vartheta(b)$$

3.3 Linear Pricing Equilibrium

When (7) holds, then a linear pricing equilibrium exists, it is unique and it is described by (3) and $x'_1 = (b-1)x'_2$. Once established existence and uniqueness, turn once more to the economic properties of equilibria. A well-behaved linear pricing equilibrium requires:

$$c < x'_1 < x'_2 < 1$$

As firstly pointed out by Cheung-Wang[4] this needs not always to be the case and this observation applies to the extended model as well. Define the following expression as:

$$\phi(b) = \frac{nc(b-1)f(b-1) + (b-1)^2f(b-1)[1-F(b-1)] +}{2f(1)\{nc + (b-1)[1-F(b-1)]\}} + \frac{-n[(b-1)^2f(b-1) - F(1)]}{2f(1)\{nc + (b-1)[1-F(b-1)]\}} + \frac{1}{2}$$

The relevance of $\phi(b)$ will become clear stating the main result of this section:

⁶It is however possible to fully describe the equilibria in this region once specified a functional form for the demand and the values of parameters, as we do in Section 4.

Proposition 2 *Suppose $\theta > \phi(b)$, then the linear pricing equilibrium is characterized by $c < x'_1 < x'_2 < 1$. If $\theta \leq \phi(b)$, then the linear pricing equilibrium is characterized by $c < x''_1 < x''_2 = 1$.*

Proposition 2 states that a well behaved equilibrium in which firms supply both one and two unit packets is found if $\theta > \phi(b)$ while only one-unit packets can be found on the market if $\theta \leq \phi(b)$.

The following re-formulation of $\phi(b)$ in terms of the marginal cost allows a more intuitive interpretation of the result of Proposition 2⁷:

$$c < c^* = \frac{(b-1)^2 f(b-1) - f(1)}{(b-1)f(b-1) - (2\theta-1)f(1)} - \frac{(b-1)[1 - F(b-1)]}{n}$$

When firms choose linear pricing, there exist a value c^* of the marginal cost over which the firms do not find it profitable to produce and sell two units packets.

The results obtained identify two regions in which the parameters space is split:

$$\begin{cases} \text{if } \theta \leq \phi(b) : c \geq c^* \Rightarrow q = 1 \text{ only} \\ \text{if } \theta > \phi(b) : c < c^* \Rightarrow q = 1, 2 \end{cases}$$

The intuition for the results presented is clear using the latest interpretation of $\phi(b)$: under linear pricing a threshold value for the unit cost exists below which firms find optimal to supply both one and two unit packets and above which only one unit packets are supplied.

4 Analysis of the Model

A uniform distribution of the consumers' willingness to pay, x , is assumed to provide a complete analysis of the model. The density and distribution function of the willingness to pay for a unit of product, x , are defined as:

$$f(x) = 1, \quad F(x) = x \quad \forall x \in [0, 1]$$

⁷An analogous result is obtained by Cheung-Wang[4] in the original model.

This assumption implies a linear demand function:

$$D_q(x_1, x_2, Q_q) = \begin{cases} 0 & \text{if } 0 < x < x_1 \\ x_2 - x_1 - Q_1 & \text{if } x_1 < x < x_2 \\ 1 - x_2 - Q_2 & \text{if } x_2 < x < 1 \end{cases}$$

Even though this is a fairly ‘special’ case, as underlined by Cheung-Wang[4], it allows us to illustrate the main features of the model.

Using these information, the condition $\theta \gtrless \phi(b)$ in Section 3.3 can be expressed as:

$$\theta \gtrless \frac{nc(b-1) + (b-1)^2(2-b) + nb(2-b)}{2[nc + (b-1)(2-b)]} + \frac{1}{2}$$

As Figure 1 witnesses for plausible combinations of c and n , the function $\theta = \phi(b)$ is downward sloping.

Even simpler is the case of non-linear pricing. The relation $\theta \geq \vartheta(b)$ becomes under a uniform distribution

$$\theta \geq \frac{b^2 - 4b + bc - 2c + 2}{2(cb - 2c - 1)}$$

The relation $\theta \leq \psi(b)$ does not change and it is clearly an increasing function of b .

Bringing together the results of Proposition 1 for non-linear pricing and of Proposition 2 for linear pricing in the context of uniformly distributed consumers’ types, gives rise to the partition of the parameters’ space illustrated in Table 1.

Table 1. Parameters and Types of Equilibria

	$\theta \leq \phi(b)$	$\theta > \phi(b)$
$\theta \geq \vartheta(b)$	Type 1	Type 2
$\theta \leq \psi(b)$	Type 3	Type 4

Figure 1 illustrates the partition of the parameters’ space $bO\theta$ for $n = 3$ and $c = 0.25$. We define Type 1 equilibria as the region of the parameters

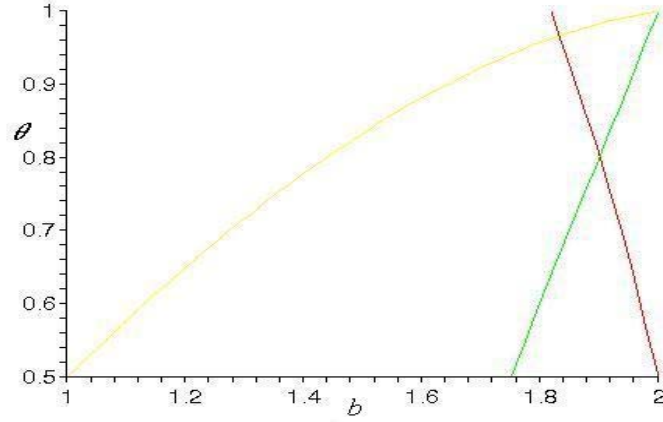


Figure 1: Equilibrium Regions for $n = 3$ and $c = 0.25$.

space characterized by firms supplying both packets under non-linear pricing and restricting to single unit size packets when linear pricing is enforced. Graphically, it is the north-western part of Figure 1. Type 2 equilibria are characterized by both packets being supplied under both non-linear and linear pricing regimes. It is the region in the north-east of Figure 1. Type 3 equilibria are remarkable in that no type of price discrimination occurs: either only the two unit packets or only the one unit packets are supplied under the different regimes. These equilibria can be identified in the south-eastern part of Figure 1. Type 4 equilibria are characterized by firms focusing only on the supply of the two-unit packets if non-linear pricing is allowed and on both sizes' packets under non-linear pricing. This equilibria take place for the combinations of parameters in the eastern part of Figure 1. Finally, it can be noticed the existence of a 'cone' of '*a priori*' non identifiable equilibria between the yellow ($\vartheta(b)$) and the green ($\psi(b)$) schedules.

Last but not least, under this specification the second order conditions for a maximum are strictly verified. The description and the analysis of the four types of equilibria follows.

4.1 Type 1 Equilibria (à la Cheung-Wang)

The first equilibrium configuration is characterized by both packets supplied under non-linear pricing (as $\theta \geq \vartheta(b)$) and only the one-unit packets under linear pricing (as $\theta \leq \phi(b)$): it constitutes the generalization of Type 1 equilibrium of Cheung-Wang[4]. The equilibrium is described by the following tables.

Table 2 reports the equilibrium values when firms practice non-linear pricing:

Table 2. Non-Linear Pricing Equilibrium of Type 1

x_1^*	$\frac{1+cn}{1+n}$
x_2^*	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^*	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i^*	$\frac{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]}{(n+1)^2(b-1)}$
CS^*	$\frac{n^2\{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]\}}{2(n+1)^2(b-1)}$

Table 3 reports the equilibrium variables in case linear pricing are chosen by all firms:

Table 3. Linear Pricing Equilibrium of Type 1

x_1''	$\frac{1+cn}{n+1}$
x_2''	\nexists
Q''	$\frac{n(1-c)}{(n+1)}$
π_i''	$\frac{(1-c)^2}{(n+1)^2}$
CS''	$\frac{n^2(1-c)^2}{2(n+1)^2}$

The comparison of equilibrium expressions allows to establish the results reported below:

Proposition 3 *Assuming the demand function is linear, in Type 1 equilibrium: (i) prices for the one-unit packets are the same under linear and non-linear pricing; (ii) total output, profits, consumer's surplus and total welfare are larger under non-linear pricing.*

4.2 Type 2 Equilibria (à la Ireland)

This case is a generalization of Ireland's benchmark: his analysis is encompassed as a special case $\theta = 1$. Under both pricing regimes all firms supply both the one and the two unit packets (as both $\theta \geq \vartheta(b)$ and $\theta > \phi(b)$ hold). This allows an important comparison with the results of Ireland[11] and a deeper analysis of the role played by size economies in this particular model and, more generally, with respect to the practice of non-linear pricing. The equilibrium expressions for the non-linear and the linear pricing case are summarized in the following tables:

Table 4. Non-Linear Pricing Equilibrium of Type 2

x_1^*	$\frac{1+cn}{1+n}$
x_2^*	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^*	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i^*	$\frac{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]}{(n+1)^2(b-1)}$
CS^*	$\frac{n^2\{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]\}}{2(n+1)^2(b-1)}$

Table 5. Linear Pricing Equilibrium of Type 2

x_1'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(n+1)}$
x_2'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(b-1)(n+1)}$
Q'	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i'	$\frac{4c+4cn+4n-8bn-6cb+2cb^2+4b^2n-6cbn+2cb^2n+2bc^2\theta^2n}{b(b-1)n(n+1)^2}$ $\frac{-2b^2c\theta-4nc\theta-4c\theta+10nbc\theta+6bc\theta-6nb^2c\theta+2n^2bc^2\theta^2}{b(b-1)n(n+1)^2}$ $\frac{-n^2b^2c^2\theta^2-2c^2bn^2\theta+c^2n^2b^2\theta-2c^2bn\theta+c^2b^2n\theta}{b(b-1)n(n+1)^2}$
CS'	$\frac{b^3+2b^3n+b^3n^2-10b^2n-b^2n^2-5b^2+b^2c^2n^2\theta^2+}{2b(b-1)(n+1)^2}$ $\frac{-4b^2n^2c\theta+16bn+8b+4bcn^2\theta-8n-4}{2b(b-1)(n+1)^2}$

The comparison of the equilibrium expressions allows to join the following conclusions:

Proposition 4 *In Type 2 equilibrium with linear demand: (i) a larger share of customers is served with one-unit packets under linear pricing while under*

non-linear pricing a larger share of two-unit packets is supplied; (ii) the total output is the same under both pricing regimes; (iii) firms' profits are always larger under non-linear pricing.

Two observations about the above results are in order. First, since the total output is constant in both equilibria, the output of each firm should be as well, i.e. $\Delta Q_i = 0$. Now, decomposing the output change in its components:

$$\Delta Q_i = 2\Delta\left(\frac{1 - x_2}{n}\right) + \Delta\left(\frac{x_2 - x_1}{n}\right) = 0$$

it can be noticed that not only the two components must have opposite signs [which was known from Point (i)] but the increase in the share of consumers served with one-unit packets under linear prices has to be twice as big as the decrease in the share of consumers buying two-units packets once firms switch from non-linear to linear pricing.

Second, the results of Ireland's original model about economy's total output and firms' profits are robust and hold in the case production displays size economies: output is constant and profits are always higher under non-linear pricing. This is not irrelevant: the results of Ireland[11] show that non-linear pricing is welfare dominated. This is because, in the linear demand case, non-linear pricing does not imply an output expansion effect.

There are no immediate analytical conclusions that can be derived for what regards the comparison between consumers' surplus and social welfare under non-linear and linear pricing. It is possible, however, to get insights from simulation evidence. Consider first the differential between consumers' surplus in the two situations:

$$\Delta CS = CS^* - CS' = \frac{[b - 2 + 2cn(\theta - 1)][(2 - b)(1 + 2n) + 2cn(\theta - 1)]}{2b(n + 1)^2}$$

This expression *a priori* can not be signed. Fixing the values of n and c at plausible levels, though, the relation ΔCS can be interpreted as a function of θ and b . Figure 2 illustrates the point: the chosen parameters are $n = 3$ and $c = 0.25$.

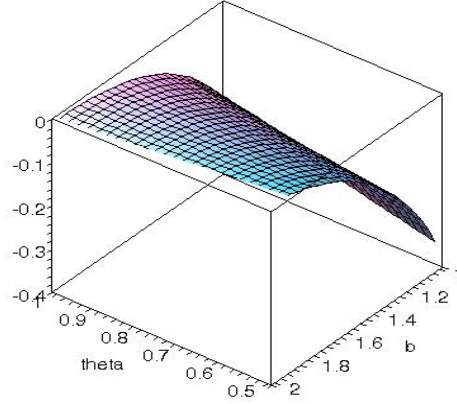


Figure 2: Consumers' Surplus Differential in Type 2 Equilibrium.

Only negative values of the function are represented in the graph while positive combinations are left blank. Inspection of the relevant region, the eastern, confirms that the function is negative for all combinations of parameters. This leads to the following remark:

Remark 1 *Ireland's conclusion that consumers are better-off if non-linear prices are prohibited is robust to the extension of the model allowing for size economies.*

The same, though, does not hold for social welfare: as Figure 3 makes clear, the gains in profits under non-linear pricing more than compensate the losses suffered by consumers and ΔW results to be positive in the relevant region defining Type 2 equilibria.

The intuition for this result, that is in contrast with Ireland's conclusions, is that non-linear pricing imposes no restrictions on firms, allowing them to be more flexible and effective in taking advantage of the cost savings deriving from size economies.

Remark 2 *In presence of size economies on the supply side, the higher flexibility allowed by non-linear pricing implies a gain in efficiency which more*

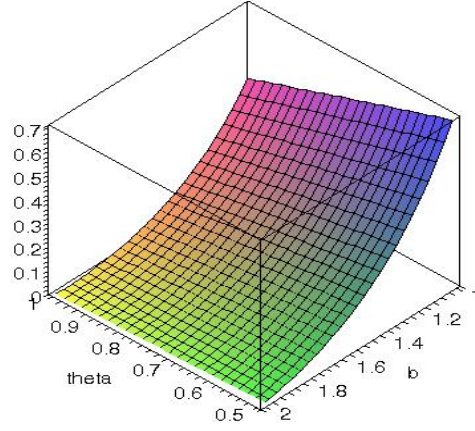


Figure 3: Social Welfare Differential in Type 2 Equilibria.

than offset the losses imposed on consumers.

4.3 Type 3 Equilibria

This case is original and peculiar at the same time: in both linear and non-linear pricing, under the given combinations of the relevant parameters, firms do not find it profitable to supply both packets. In a sense, no price discrimination exists in equilibrium under either one price regime or the other; the shape of the equilibrium is nevertheless determined by the degree of price freedom firms enjoy

Only two-units packets are on the market under non-linear pricing as $\theta \leq \psi(b)$ and as the following table describing this equilibrium makes clear:

Table 6. Non-Linear Pricing Equilibrium of Type 3

x_1^{**}	$\frac{b}{2}$
x_2^{**}	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^{**}	$\frac{2n[(b-1)-(2\theta-1)c]}{(b-1)(n+1)}$
π_i^{**}	$\frac{[(b-1)-(2\theta-1)c](b^2-b-bcn-2\theta cb+2\theta c+2\theta cn)}{(n+1)^2(b-1)^2}$
CS^{**}	$\frac{bn^2[(b-1)-(2\theta-1)c]^2}{2(n+1)^2(b-1)^2}$

whereas, only one-unit packets are offered under linear pricing as $\theta \leq \phi(b)$ and as it can be seen in the table below.

Table 7. Linear Pricing Equilibrium of Type 3

x_1''	$\frac{1+cn}{n+1}$
x_2''	\nexists
Q''	$\frac{n(1-c)}{(n+1)}$
π_i''	$\frac{(1-c)^2}{(n+1)^2}$
CS''	$\frac{n^2(1-c)^2}{2(n+1)^2}$

By comparing the equilibrium expressions, the following results are obtained:

Proposition 5 *Assuming the demand function is linear, in Type 3 equilibrium: (i) the share of consumers served with two units under non-linear pricing is larger than the share supplied with one unit packets under linear pricing; (ii) total output is larger under non-linear pricing.*

Unfortunately, no conclusive results are available with respect to profits, consumers' surplus and total welfare. Using again the strategy of fixing plausible values of the parameters c and n , a few hints on the comparison between non-linear and linear prices can be given.

Figure 4 represents $\Delta\pi$, the differential between profits under non-linear and linear pricing, for plausible combinations of parameters. The relevant region for this case is the southern part: for $n = 3$ the function is mainly positive, apart for the half moon shaped region in the south-eastern part of the figure, representing rather extreme combinations of parameters. The negative area expands as n increases⁸: a general conclusion, then, can not be reached.

The uncertainty over the profit effects of non-linear pricing however allows to highlight a very interesting mechanism related to the oligopolistic nature

⁸The results of simulations for different values of n and c are available upon request from the author.

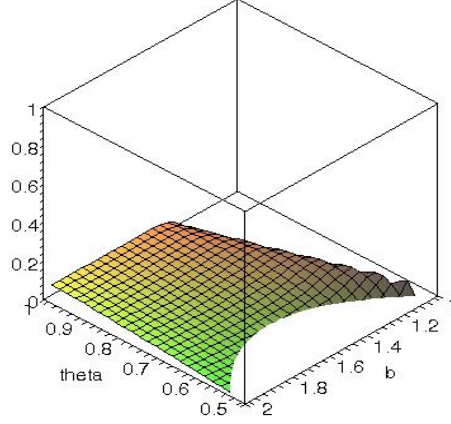


Figure 4: Profit Differential in Type 3 Equilibria.

of the model. The model encompasses a monopolistic market structure. The only profit maximizing firm should always be able to do at least as well using non-linear pricing: that strategy, in fact, encompasses linear pricing in the limit. This needs not to be the case as the number of firms increases. The reason why non-linear pricing may be dominated by linear pricing is related to very last nature of firms' maximizand function: according to Bergstrom-Varian[3] this is a combination of aggregate consumer surplus and aggregate profits. Following that approach, Ireland[11] shows that the function firms maximize, $G(Y)$ can be written as:

$$G(Y) = \frac{(n-1)CS + n\pi}{n} \quad (8)$$

This specification, *mutatis mutandis*⁹, is robust to the extended version of the model considered here. Ireland[11] insight is that, as $G(Y)$ is maximized freely in a non-linear pricing equilibrium and $G(Y)$ is maximized under constraints in the linear pricing equilibrium, then necessarily needs the following

⁹The expressions of aggregate consumers surplus and profit are different in this case: this does not affect the validity of the analysis.

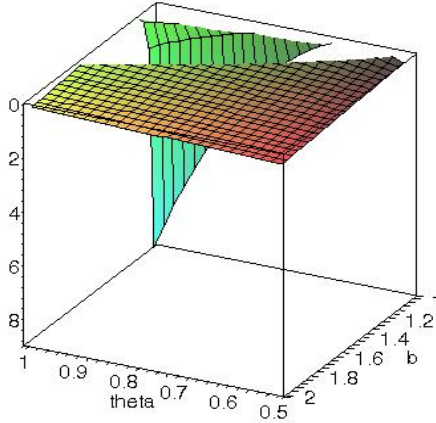


Figure 5: Consumers' Surplus Differential in Equilibria of Type 3.

needs to hold:

$$G(Y^{NLP}) > G(Y^{LP})$$

assuming the constraint is strictly binding. As the consumer surplus is always higher in a linear pricing equilibrium, equation (8) implies that profit must be higher in the non-linear pricing one. As shown by Figure 5, in the case analyzed here consumers surplus is always *higher* in a non-linear pricing equilibrium, so the relation between profits π^* and π' can go either way, without this fact compromising the logical consistency of the model.

The intuition behind this result is that strategic interaction may determine all firms ending up in a sub-optimal outcome: this is due to the externality they exercise on each other when maximizing their profits.

In conclusion, passing from a linear to a non-linear pricing equilibrium, the positive effects for the consumers more than offset the negative effects, if any, on profits. Simulations as in Figure 5 and 6 highlight the positive effect of non-linear pricing on consumers' surplus and welfare. The latter result is clearly driven by the expansion in total output guaranteed by price

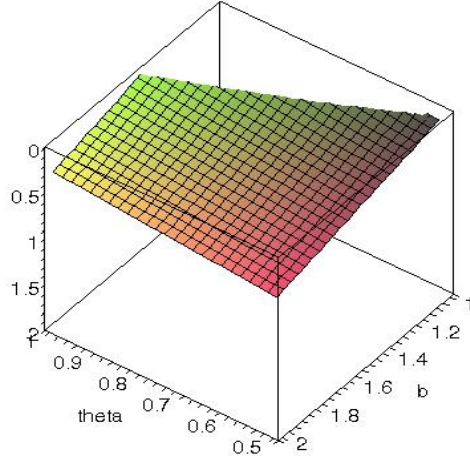


Figure 6: Social Welfare Differential in Type 3 Equilibria.

discrimination.

This result can be contrasted with the received literature: while output expansion under non-linear pricing is often achieved by expanding the share of customers served, in this case output increases despite firms are serving an identical share of demand under both regimes. To summarize:

Remark 3 *In Type 3 equilibria output is expanded under non-linear pricing even maintaining constant the share of consumers served. This has positive effects for both consumers and welfare.*

4.4 Type 4 Equilibria

In this situation firms supply only two-unit packets under non-linear pricing since the condition $\theta \leq \psi(b)$ holds. Under linear pricing, nevertheless both one and two-unit packets are supplied as the relation $\theta > \phi(b)$ is verified. The analysis of this type of equilibrium is original since the outcomes arising are characteristic only of the size economies setting.

The expressions characterizing the symmetric non-cooperative Cournot equilibrium under non-linear pricing are summarized in the following table:

Table 8. Non-Linear Pricing Equilibrium of Type 4

x_1^{**}	\nexists
x_2^{**}	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^{**}	$\frac{2n[(b-1)-(2\theta-1)c]}{(b-1)(n+1)}$
π_i^{**}	$\frac{[(b-1)-(2\theta-1)c](b^2-b-bcn-2\theta cb+2\theta c+2\theta cn)}{(n+1)^2(b-1)^2}$
CS^{**}	$\frac{bn^2[(b-1)-(2\theta-1)c]^2}{2(n+1)^2(b-1)^2}$

The symmetric equilibrium when linear prices are practiced is characterized by the expressions reported in Table 9:

Table 9. Linear Pricing Equilibrium of Type 4

x_1'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(n+1)}$
x_2'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(b-1)(n+1)}$
Q'	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i'	$\frac{4c+4cn+4n-8bn-6cb+2cb^2+4b^2n-6cbn+2cb^2n+2bc^2\theta^2n}{b(b-1)n(n+1)^2}$ $\frac{-2b^2c\theta-4nc\theta-4c\theta+10nbc\theta+6bc\theta-6nb^2c\theta+2n^2bc^2\theta^2}{b(b-1)n(n+1)^2}$ $\frac{-n^2b^2c^2\theta^2-2c^2bn^2\theta+c^2n^2b^2\theta-2c^2bn\theta+c^2b^2n\theta}{b(b-1)n(n+1)^2}$
CS'	$\frac{b^3+2b^3n+b^3n^2-10b^2n-b^2n^2-5b^2+b^2c^2n^2\theta^2+}{2b(b-1)(n+1)^2}$ $\frac{-4b^2n^2c\theta+16bn+8b+4bcn^2\theta-8n-4}{2b(b-1)(n+1)^2}$

The analysis of the equilibrium expressions leads to the following results:

Proposition 6 *In Type 4 equilibrium with linear demand: (i) a larger share of customers is served with two-unit packets under non-linear pricing than under linear; (ii) total output is larger under non-linear pricing; (iii) the price of two unit-packets is higher under non-linear pricing; (iv) profits are larger under non-linear pricing than that under the linear pricing scheme.*

The most striking feature of this equilibrium situation is that, despite the fact that firms supply both the one and two unit packets under linear-pricing,

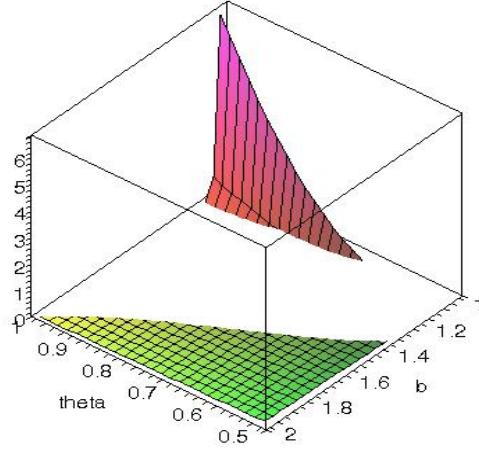


Figure 7: Consumers' Surplus Differential in Type 4 Equilibria.

total output is larger under non-linear pricing: this is due to the fact that a larger share of two-unit packets is supplied under non-linear pricing and this effect is not compensated by the one-unit packets that are offered only under linear pricing.

Furthermore, the price of two-unit packets under linear pricing is at most as big as the one registered under non-linear pricing: this, in turn, implies that firms' profits are greater in the latter situation. This is an important result that can be interpreted as follows: when size economies are so intense that firms find it convenient to produce only two-unit packets, the cost savings allow to serve a larger share of consumers, which also results to be more profitable than serving both sizes under linear pricing.

No definitive results can be obtained on consumers' surplus and total welfare, but some considerations can be drawn by comparing the equilibrium expressions. Considering again the consumers' surplus differential between non-linear and linear pricing, Figure 7 displays the results for $c = 0.25$ and $n = 3$. The function ΔCS is positive for the relevant combinations of θ and b , the south-western region in the picture.

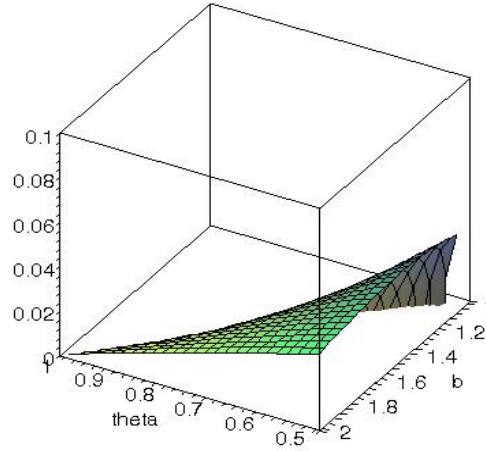


Figure 8: Social Welfare Differential in Type 4 Equilibria.

As a result, geometrical considerations seem to suggest not only that non-linear prices can leave consumers better-off but that this is a very likely case. If this is the case, a non-linear pricing equilibrium Pareto-dominates the linear pricing one. This is witnessed by Figure 8 that reports the graph of $\Delta W = W^{**} - W'$ for the same values of n and c : the function is clearly positive in the relevant region.

The following remark summarizes the results regarding these equilibria:

Remark 4 *Simulations evidence suggests that total welfare is higher when switching from linear to non-linear pricing. For the relevant combinations of parameters consumers are better-off, implying that non-linear pricing is Pareto-superior to linear pricing.*

5 Conclusions

This paper has provided two main contributions. First, it tackled the issue of size economies in the context of non-linear pricing within a model

of oligopoly. The relation between the welfare effects of second degree price discrimination and size or scale economies on the supply side is a debated topic in the literature. The paper provides an inquiry on the relationship between the demand and the cost structure of the market. Second, conditions under which firms find it optimal to supply packets of different size in equilibrium, both under non-linear and linear pricing, are identified. The main result can be summarized as follows: the number of packets of product supplied in equilibrium depends on the relative intensity of size economies, both under non-linear and linear pricing.

Non-linear pricing in presence of a technology characterized by size economies is likely to be welfare enhancing. However, not only non-linear pricing can be socially preferred to linear pricing, cases in which non-linear pricing is Pareto-superior to linear pricing are devised¹⁰. Generalizing Cheung-Wang[4] findings this is the case of Type 1 equilibria¹¹; evidence from simulations suggests that this is a concrete possibility also in Type 3 and Type 4 equilibria.

The main *caveat* of the analysis is, then, that a policy prescription to forbid firms to practice non-linear pricing needs a careful qualification. The relevant features which should be taken into account in evaluating the merits of non-linear pricing as a market practice are: 1) the possibility of serving under non-linear pricing shares of customers that otherwise would be excluded from the market; 2) the technological structure of the firms active in the sector, in order to assess the relation between the industry's technology and the impact of non-linear pricing on social welfare.

The analysis can be further extended in a number of ways. The model easily and readily extends to the case in which the packets supplied is generically m . Generality was traded-off to leave space to the more intuitive two-packets case.

Less tractable is the analysis of asymmetries between firms in this model.

¹⁰Pareto-superiority in this context is used meaning that both firms and customers are better-off.

¹¹A further generalization of this result can be found in Appendix B.

There are several interesting issues when dealing with asymmetric settings. One of this is: what happens if one of the firms is more efficient than the rivals in taking advantage of size economies? Even more interesting, perhaps, is the analysis of the effect of an asymmetric choice of strategies. Suppose the market is a duopoly and firms are free to choose their pricing policy. What would be the outcome in the case of one firm practicing non-linear pricing and the other adopting a linear price schedules? This exercise would allow one to characterize completely the game faced by firms. In several duopolistic contexts, the ability to price-discriminate is privately profitable but leads in equilibrium to a less preferred outcome¹². Despite the evaluation of the conditions under which firms face such a game may be important, it is not technically immediate to achieve. Two further possible extensions are worth being mentioned. First, a comparison of the results with a setting in which firms compete *à la Bertrand* may be of interest. Moreover, the analysis can be extended to the case of production technology characterized by negative returns or the more extreme case of ‘damaged goods’.

A final word deserves the quality interpretation that can be given to the model. If the good is supplied in different qualities, instead of packets, several real-world economic situations can be interpreted through this model. A related paper[15] uses a similar approach to shed light on the regulation of broadband internet and network neutrality regulation. Moreover, the approach of this paper have many common features with the recent theoretical analysis of vertical quality differentiation: a productive parallel between these streams of literature may improve knowledge on both topics.

¹²Thisse-Vives[18] find this result in a spatial context, Fudenberg-Tirole[10] in the case of dynamic pricing and Corts[5] with respect to third-degree price discrimination are a few important examples.

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A Proof of Propositions 1-6

This appendix contains the proofs of Proposition 1 to Proposition 6.

Proof of Proposition 1 It is firstly established that firms do not price below marginal cost. To prove that $x_1^* > c$ and $x_2^* > \frac{(2\theta-1)c}{(b-1)}$ notice that x_1^* and x_2^* are defined respectively by $FOC_{NLP1}(x) = 0$ and $FOC_{NLP2}(x) = 0$. If second order conditions (5)-(6) are met, (1)-(2) are also monotonically decreasing in x . Now, since $FOC_{NLP1}(c) = \frac{1-F(c)}{n} > 0$ and $FOC_{NLP2}(\frac{(2\theta-1)c}{b-1}) = \frac{1-F[\frac{(2\theta-1)c}{(b-1)}]}{n} > 0$, the claim is verified.

It is then to be proved that if $\theta \geq \vartheta(b)$ then $x_1^* < x_2^*$, consider the function:

$$\Omega(x) = FOC_{NLP2}(x) - FOC_{NLP1}(x)$$

for a generic, given x . Notice first that by assumptions and by (5) - (6), both $FOC_{NLP1}(x)$ and $FOC_{NLP2}(x)$ are continuous and weakly decreasing in x . Moreover, it can be shown that $FOC_{NLP1}(x)$ is decreasing at a higher rate than $FOC_{NLP2}(x)$; having defined $\Xi(x) = SOC_{NLP2}(x) - SOC_{NLP1}(x)$ it is verified that: $\Xi(x) > 0 \quad \forall x \in [0, 1]$.

By algebraic manipulations, it is found that $\Omega(x)|_{x=\frac{2\theta-1}{b-1}c} \geq 0 \Leftrightarrow \theta \geq \vartheta(b)$. As $FOC_{NLP1}(x)$ is decreasing at a faster rate, this is sufficient to ensure that $x \text{ s.t. } \{FOC_{NLP2}(x) = 0\} > x \text{ s.t. } \{FOC_{NLP1} = 0\}$ which is equivalent to say $x_2^* > x_1^*$.

It can also be derived that: $\Omega(x)|_{x=1} \leq 0 \Leftrightarrow \theta \leq \psi(b)$. This a sufficient condition to ensure that the equilibrium collapses to $x_1^{**} \equiv x_2^{**}$ *Q.E.D.*

Proof of Proposition 2 The first inequality ($c < x'_1$) is showed to hold by checking that $FOC_{LP}[c/(b-1)] > 0$ and noticing that also $FOC_{LP}(x)$ is monotonically decreasing in x . The second inequality ($x'_1 < x'_2$) is verified by definition. To see that the last inequality holds notice that $x'_2 < 1$ if and

only if $FOC_{LP}(1) < 0$ which requires:

$$FOC_{LP}(1) = -(b-1)[(b-1)-c]f(b-1) - f(1)[(b-1) - (2\theta-1)c] + \\ + \frac{(b-1)}{n}[1 - F(b-1)] < 0$$

This inequality can be expressed as a relation between θ and b :

$$\theta > \frac{nc(b-1)f(b-1) + (b-1)^2f(b-1)[1 - F(b-1)] +}{2f(1)\{nc + (b-1)[1 - F(b-1)]\}} \\ \frac{-n[(b-1)^2f(b-1) - F(1)]}{2f(1)\{nc + (b-1)[1 - F(b-1)]\}} + \frac{1}{2} = \phi(b)$$

$x_2'' \equiv 1$ in case $\theta \leq \phi(b)$ *Q.E.D.*

Proof of Proposition 3 Point (i) follows from: $p_1^* = x_1^*$ and $p_1'' = x_1''$. As $x_1^* = x_1''$, the result is immediately proven. Turning to point (ii), $x_1^* = x_1''$ implies that the same share of consumers is served under both pricing regimes. It should be noticed under non-linear pricing, a share $1 - x_2^*$ of customers consumes the two-units packet then, in equilibrium, the output must be higher under non-linear pricing. As $p_1^* = p_1''$ and the total output is higher, profits must be higher under non-linear pricing. This is confirmed by algebraical comparison of π_i^* and π_i'' . A similar result is obtained by directly comparing the consumers' surplus in the two situations. The result regarding total welfare then follows by definition. *Q.E.D.*

Proof of Proposition 4 Point (i) descends from direct comparisons of the equilibrium expressions for the choice variables. First of all, $\Delta x_2 = x_2^* - x_2' \leq 0$ and is equal zero only in the extreme case $b = 1$. Furthermore, once again by direct comparison, it is obtained $\Delta x_1 = x_1^* - x_1' > 0$ under the assumptions made on the parameters. These results imply that the share of

customers served with two-units packets is (weakly) larger under non-linear pricing:

$$1 - x_2^* \geq 1 - x_2'$$

while, under linear pricing, is larger the share of consumers buying the one-unit packet:

$$x_2^* - x_1^* < x_2' - x_1'$$

Point (ii) is immediate by looking at Q^* and Q' . Point (iii) comes from observing that $\Delta\pi_i = \pi_i^* - \pi_i' > 0$ for all the combinations of the relevant parameters. *Q.E.D.*

Proof of Proposition 5 Since, by direct comparison, $x_1'' \geq x_2^{**}$ it is immediate that both Point (i) and (ii) are verified. *Q.E.D.*

Proof of Proposition 6 Point (i) follows by direct comparison: $\Delta x_2 = x_2^{**} - x_2' \leq 0$ and it is equal to zero only in the special case $b = 2$ and $\theta = 1$. From this, it is immediate to see that $1 - x_2^{**}$ is larger than $1 - x_2'$.

Direct comparison and the restriction $\theta \leq \psi(b)$ permit to show that $\Delta Q = Q^{**} - Q' \geq 0$ for all the feasible combinations of the parameters, so that the output under non-linear pricing results larger or at least equal to the one under linear prices. Point (iii) is proved by showing, through simple algebraic manipulations, that $\Delta p_2 = p_2^{**} - p_2' > 0$. Point (iv) is derived by observing that $Q^{**} \geq Q' = Q_1' + Q_2'$ where Q_1' and Q_2' represent the contribution of one- and two-units packets, respectively, to the total output; furthermore, since $p_2' - 2\theta c = 2(p_1' - \theta c) > p_1 - c$ for all $\theta \in [0, 1)$ then, *a fortiori*, $p_2^{**} - 2\theta c > p_1' - c$. Jointly these observations imply that the total profits are:

$$\Pi^{**} = (p_2^{**} - 2\theta c)Q^{**} > (p_2' - 2\theta c)Q_2' + (p_1' - c)Q_1' = \Pi'$$

which, by dividing by n , gives the wanted result for the firms' profits. *Q.E.D.*

B Type 1 Equilibrium: A General Result

The results stated in Proposition 3 are just a corollary of Proposition 7, which addresses the general case. As illustrated in what follows these results imply that if the parameters of demand and cost function are such that under linear pricing it is optimal to supply only one-unit packets, then firms have profit advantages to practice non-linear prices. Furthermore this also drives to a Pareto-superior equilibrium: both firms and consumers are better off in the latest situation. A step backwards to the general specification of Type 1 equilibrium is necessary at this stage and its description is provided.

The non-linear pricing equilibrium is:

$$\begin{aligned} Q^{NLP} &= 2 - F(x_2^*) - F(x_1^*) \\ \pi_i^{NLP} &= (x_1^* - c) \frac{F(x_2^*) - F(x_1^*)}{n} + [(b-1)x_2^* + x_1^* - 2\theta c] \frac{1 - F(x_2^*)}{n} \\ CS^{NLP} &= \int_{x_1^*}^{x_2^*} (x - x_1^*) f(x) dx + \int_{x_2^*}^1 \{bx - [(b-1)x_2^* + x_1^*]\} f(x) dx \end{aligned}$$

where:

$$x_1^* \text{ s.t. } \frac{1 - F(x_1^*)}{n} - f(x_1^*)(x_1^* - c) = 0$$

$$x_2^* \text{ s.t. } (b-1) \frac{1 - F(x_2^*)}{n} - f(x_2^*)[(b-1)x_2^* - (2\theta - 1)c] = 0$$

and: $\theta > \psi(b)$.

The linear pricing equilibrium is:

$$\begin{aligned} Q^{LP} &= 1 - F(x_1'') \\ \pi_i^{LP} &= (x_1'' - c) \frac{1 - F(x_1'')}{n} \\ CS^{LP} &= \int_{x_1''}^1 (x - x_1'') f(x) dx \end{aligned}$$

where $x_1'' = (b-1)x_2''$ and $x_2'' > 1$ is the solution to (3) as $\theta < \phi(b)$.

Proposition 7 states the relations between non-linear and linear pricing equilibrium:

Proposition 7 *Equilibria of Type 1, when $\theta > \psi(b)$ and $\theta \leq \phi(b)$, are characterized by:*

1.

$$x_1^* \equiv x_1'' \quad \forall x \in [0, 1] \quad (9)$$

2.

$$Q^{NLP} \geq Q^{LP} \quad \forall x \in [0, 1]$$

3.

$$\pi^{NLP} \geq \pi^{LP} \quad \forall x \in [0, 1]$$

4.

$$CS^{NLP} \geq CS^{LP} \quad \forall x \in [0, 1]$$

Proof Point 1. derives by simply observing that x_1^* and x_1' are identified by the same first order condition: this, in turn, implies they coincide. Point 2. and 3. are direct implications of the result in 1. while 4. is the result of direct comparison between the equilibrium expressions of the consumers' surplus under non-linear and linear pricing. *Q.E.D.*

This is a general and strong result: equation (9) allows to conclude that the marginal customer choosing to consume one unit is the same under non-linear and linear pricing. This obviously implies that the total output is always larger under non-linear pricing, under which both one and two unit packets are supplied. Moreover and more importantly, not only firms' profits but also consumers' surplus are always higher under non-linear pricing in this case.

These conclusions are the exact generalization of Proposition 6 in Cheung-Wang[4], that proves to be robust to the extension involving size economies. As the authors recall, the result stated parallels a very well known result concerning third degree price discrimination: this practice can be welfare enhancing in situations in which it allows to serve a share of demand that would stay out of the market if a uniform price were practiced.